O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

ECEN/MAE 5513
Stochastic Systems
Fall 2011
Midterm Exam #2

PLEASE DO ALL FIVE PROBLEMS

Name : ______________________________

E-Mail Address:_______________________________________

Problem 1:

Show that the joint distribution function of random variables *X* and *Y*, conditioning on the event $B = \{y_a < Y \le y_b\}$ is

$$F_{X,Y}(x, y \mid y_a < Y \le y_b) = \begin{cases} 0, & y \le y_a \\ \dfrac{F_{X,Y}(x,y) - F_{X,Y}(x,y_a)}{F_Y(y_b) - F_Y(y_a)}, & y_a < y \le y_b \\ \dfrac{F_{X,Y}(x,y_b) - F_{X,Y}(x,y_a)}{F_Y(y_b) - F_Y(y_a)}, & y_b < y \end{cases},$$

and find the corresponding joint density function conditioning on the same event *B* as $f_{X,Y}(x, y \mid y_a < Y \le y_b)$.

Problem 2:

Three statistically independent random variables X_1, X_2 and X_3 are defined by

$$\overline{X}_1 = -1, \sigma_{X_1}^2 = 2.0, \overline{X}_2 = 0.6, \sigma_{X_2}^2 = 1.5, \text{ and } \overline{X}_3 = 1.8, \sigma_{X_3}^2 = 0.8 \ .$$

Write the equation describing the Gaussian approximation for the density function of the sum $X = X_1 + X_2 + X_3$.

<u>Problem 3</u>:

The *zero-mean* and *unit-variance Gaussian* random variables X and Y are statistically independent. Find the probability density function of the random variable $W = \sqrt{X^2 + Y^2}$.

Problem 4:

Two ransom variables X and Y are related by the expression

$$Y = aX + b,$$

where a and b are any real numbers.

a) Show that their correlation coefficient is

$$\rho = \begin{cases} 1, & \text{if } a > 0 \text{ for any } b \\ -1, & \text{if } a < 0 \text{ for any } b \end{cases}$$

b) Show that their covariance is

$$C_{XY} = a\sigma_X^2,$$

where σ_X^2 is the variance of X.

<u>Problem 5</u>:

Suppose the annual snowfalls (accumulated depths in meters) for two nearby alpines ski resorts are adequately represented by jointly Gaussian random variables *X* and *Y*, for which $\rho = 0.82$, $\sigma_X = 1.5m$, $\sigma_Y = 1.2m$, and $R_{XY} = 81.476m^2$. If the average snowfall at one resort is 10*m*, what is the average at the other resort?